UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Ordinary Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

**MATHEMATICS (SYLLABUS D)** **4024/21**

Paper 2 **May/June 2012**

**2 hours 30 minutes**

Candidates answer on the Question Paper.

Additional Materials: Geometrical instruments
Electronic calculator

**READ THESE INSTRUCTIONS FIRST**

Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

**Section A**
Answer **all** questions.

**Section B**
Answer any **four** questions.

If working is needed for any question it must be shown in the space below that question.
Omission of essential working will result in loss of marks.
You are expected to use an electronic calculator to evaluate explicit numerical expressions.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For $\pi$, use either your calculator value or 3.142, unless the question requires the answer in terms of $\pi$.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 100.

| For Examiner's Use |
| --- |
| |

This document consists of **24** printed pages.



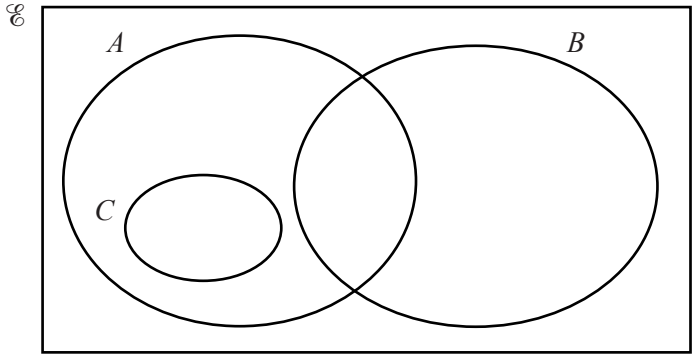

UNIVERSITY of CAMBRIDGE International Examinations

**Section A** [52 marks]

Answer **all** questions in this section.

*Do not write in this margin*

**1** **(a)** The sets $A$, $B$ and $C$ are shown in the Venn diagram.




$\mathscr{E} = \{ x : x \text{ is an integer, } 1 \leqslant x \leqslant 18 \}$
$A = \{ x : x \text{ is an even number} \}$
$B = \{ x : x \text{ is a multiple of 5} \}$

**(i)** Find $\text{n}(A \cup B)$.

*Answer* .....[1]

**(ii)** **(a)** Given that $A \cap B' \cap C' = \{2, 6, 14, 18\}$, list the members of $C$.

*Answer* .....[1]

**(b)** Describe the set $C$ in words.

*Answer* $C = \{ x : x \text{ is}$ ..... $\}$ [1]

**(b)** A school offers piano lessons and flute lessons to a group of 50 children.

*Do not write in this margin*

Of these children, 28 attend piano lessons
17 attend flute lessons
12 attend neither piano lessons nor flute lessons.

By drawing a Venn diagram, or otherwise, find the number of children who attend only the piano lessons.

*Answer* ..... [2]

**2** **(a)** Sunil needs to hire a digger from Monday to Thursday one week and on Monday and Tuesday the following week.
The Hire company charges $48 each time the digger is hired plus $13 per day.
He has two options.

Option 1: Hire the digger for four days, return it and then hire it again for two days.

Option 2: Hire it continuously from the first Monday to the second Tuesday.

Which is the cheaper option and by how much?

*Answer* Option ..... is cheaper by $ ..... [2]

**(b)** Tina invests some money in an account that earns simple interest at 3% per year.
At the end of one year the investment is worth $2781.

How much money did she invest?

*Answer* $..... [2]

**3** **(a)** Factorise $9x^2 - 64y^2$.

*Do not write in this margin*

*Answer* .....[1]

**(b)** The product of three numbers 4, $x$ and $(x + 3)$ is 55.

Form an equation in $x$ and solve it to find the possible values of $x$.

*Answer* $x =$ ..... or .....[3]

**(c)** **(i)** Given that $\frac{x-1}{3} - \frac{5}{x+2} = 1$ show that $x^2 - 2x - 23 = 0$.

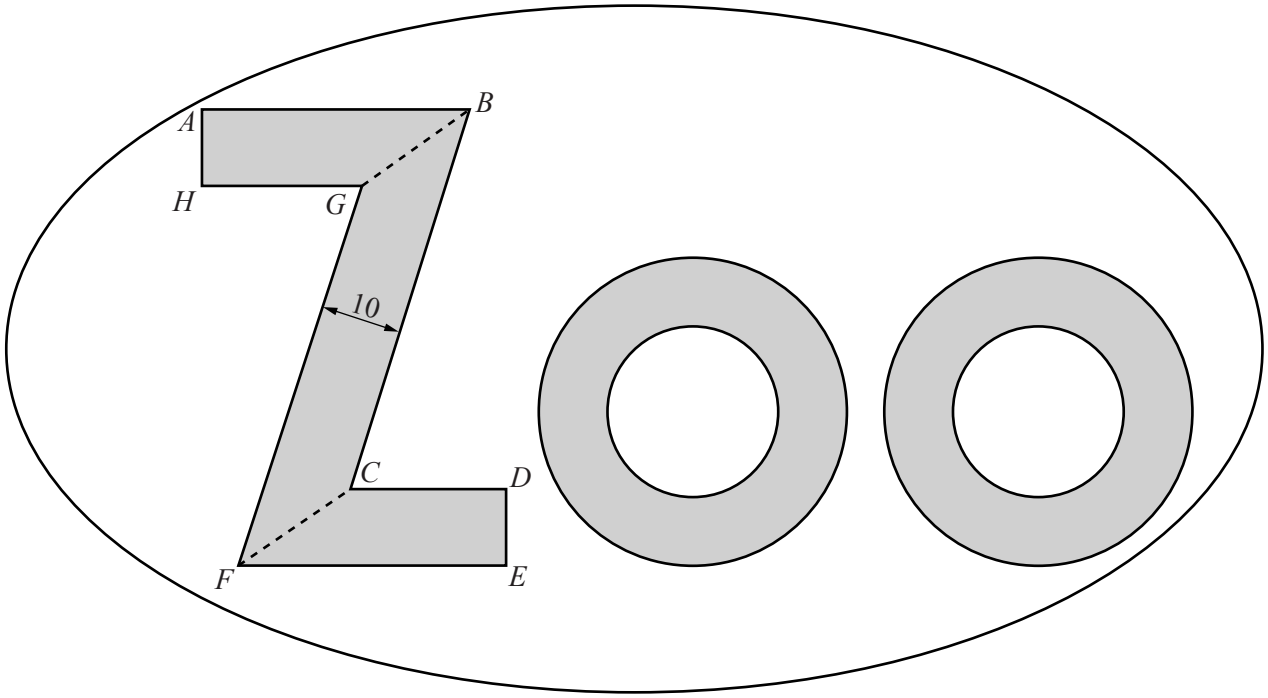
[2]

**(ii)** Solve $x^2 - 2x - 23 = 0$.
Give your answers correct to one decimal place.

*Answer* $x =$ ..... or .....[3]

**4** The entrance to a zoo has this sign above it.

*Do not write in this margin*




**(a)** The letter Z has rotational symmetry order 2 and $DE$ is perpendicular to $FE$ and $CD$.
$CD = 35$ cm, $FE = 50$ cm, $DE = 10$ cm and $BC = 81$ cm.
The perpendicular distance between $BC$ and $GF$ is 10 cm.

Calculate the area of the letter Z.

*Answer* ..... $cm^2$ [3]

**(b)** The shaded area of one letter O is 1206 $cm^2$.
The radius of the unshaded inner circle is 15 cm.

Calculate the radius of the outer circle.

*Answer* ..... cm [3]

**(c)** The sign above the exit of the zoo is geometrically similar to the one above the entrance.
The radius of the inner circle of the letter O on the sign above the exit is 10 cm.

**(i)** The length of the base of the letter Z on the sign above the entrance is 50 cm.

Calculate the length of the base of the letter Z on the sign above the exit.

*Answer* ..... cm [1]

**(ii)** The area of the sign above the entrance is $A$ $cm^2$.
The area of the sign above the exit is $kA$ $cm^2$.

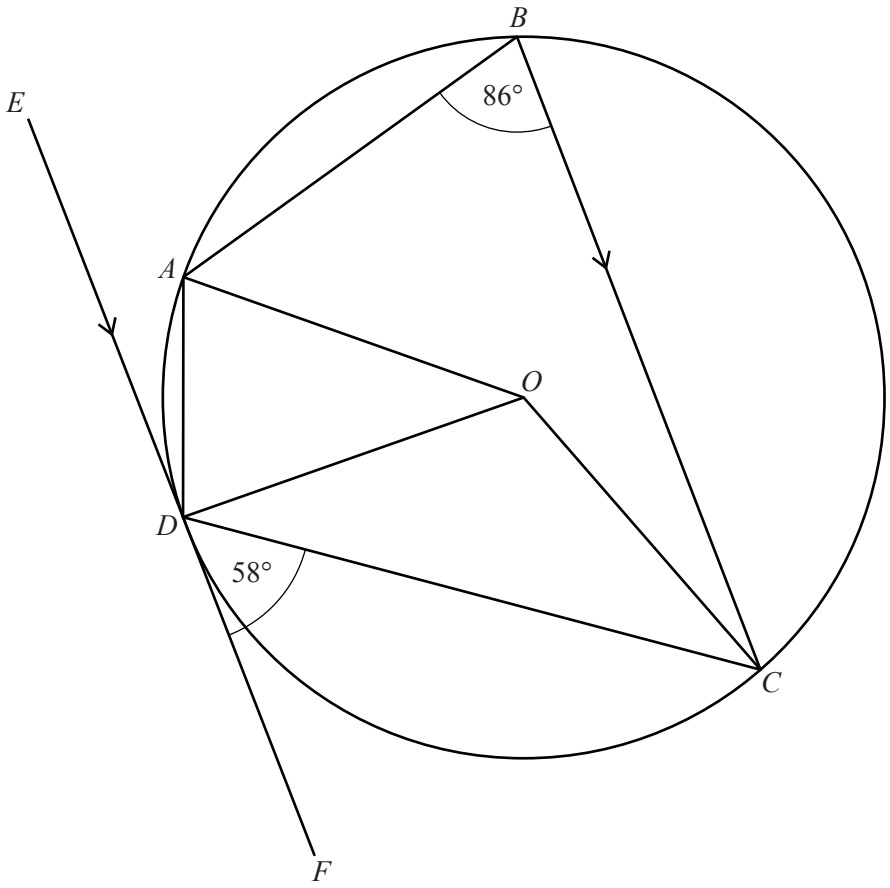
Write down the value of $k$ as a fraction in its simplest form.

*Answer* ..... [2]

*Do not write in this margin*

**5**






$A$, $B$, $C$ and $D$ are points on the circumference of a circle, centre $O$.
$EF$ is the tangent to the circle at $D$ and is parallel to $BC$.
$A\hat{B}C = 86°$ and $C\hat{D}F = 58°$.

**(a)** Find $O\hat{D}C$.

*Answer* .....[1]

**(b)** Explain why $O\hat{C}B = 26°$.

*Answer* .....

.....[2]

**(c)** Find

*Do not write in this margin*

**(i)** $A\hat{D}C$,

*Answer* .....[1]

**(ii)** $A\hat{D}E$,

*Answer* .....[1]

**(iii)** $A\hat{O}D$,

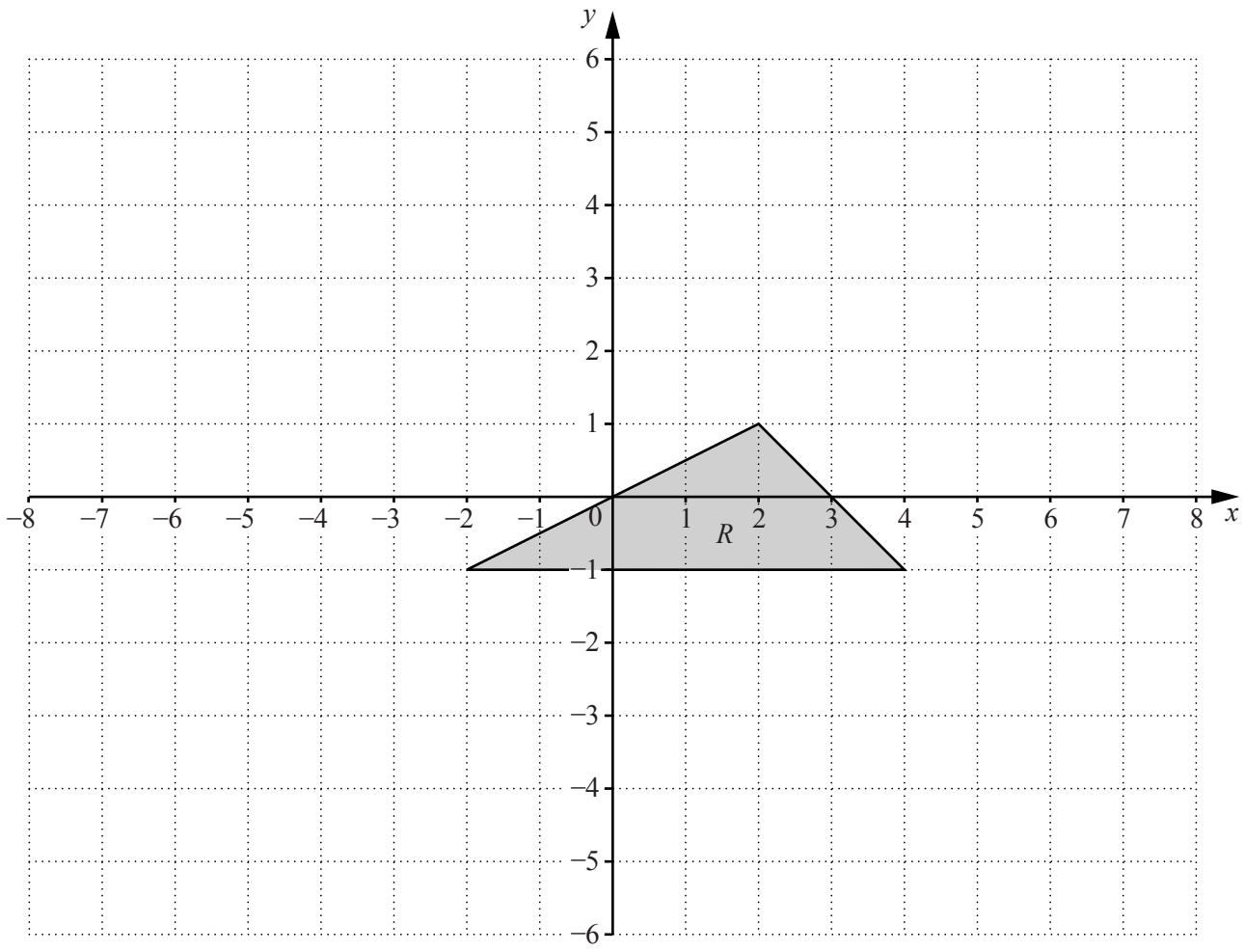
*Answer* .....[1]

**(iv)** $B\hat{A}O$.

*Answer* .....[1]

**6**






Triangle $R$ has vertices (–2, –1), (2, 1) and (4, –1).

**(a)** The gradients of the sides of triangle $R$ are 0, –1 and $k$.

Find $k$.

*Answer* .....[1]

**(b)** One of the inequalities that defines the shaded region is $x + y \leqslant 3$.

Write down the other two inequalities that define this region.

*Answer* .....

.....[2]

**(c)** Triangle $R$ is mapped onto triangle $P$ by a reflection in the line $y = -2$.

Draw and label triangle $P$. [2]

*Do not write in this margin*

**(d)** Triangle $R$ is mapped onto triangle $Q$ by a stretch where the invariant line is the $y$-axis. This transformation maps the vertex (2, 1) onto (4, 1).

**(i)** For this stretch, state the scale factor.

*Answer* ..... [1]

**(ii)** Find the coordinates of the vertex (4, –1) when it is transformed by this stretch.

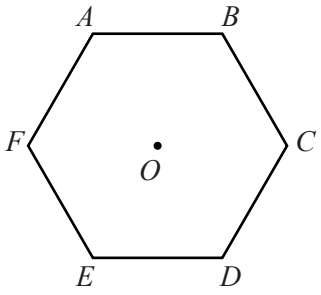
*Answer* (..... , .....) [1]

**(iii)** Find the area of triangle $Q$.

*Answer* ..... units$^2$ [2]

**7** *ABCDEF* is a regular hexagon with centre *O*.






**(a) (i)** Find $A\hat{O}B$.

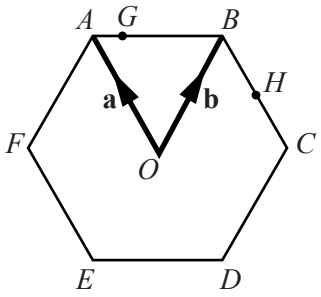
*Answer* .....[1]

**(ii)** Explain why $AO = BC$.

*Answer* .....[1]

**(b)** $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.

*G* is the point on *AB* such that *AG* : *GB* is 1 : 3.
*H* is the midpoint of *BC*.




Express, as simply as possible, in terms of **a** and **b**,

**(i)** $\overrightarrow{AB}$,

*Answer* .....[1]

**(ii)** $\overrightarrow{FB}$,

*Answer* .....[1]

**(iii)** $\overrightarrow{OG}$,

*Do not write in this margin*

*Answer* .....[2]

**(iv)** $\overrightarrow{OH}$,

*Answer* .....[1]

**(v)** $\overrightarrow{GH}$.

*Answer* .....[2]

---

**Section B** [48 marks]

Answer **four** questions in this section.

Each question in this section carries 12 marks.

*Do not write in this margin*

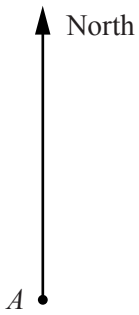
**8** **(a)** Three towns, $A$, $B$ and $C$, are located such that $AB = 90$ km, $BC = 100$ km and $AC = 85$ km. The bearing of $B$ from $A$ is $127^o$.

**(i)** Write down the bearing of $A$ from $B$.

*Answer* ..... [1]

**(ii)** Of the three towns, $A$ is the furthest north.

Using a scale of 1 cm to 10 km, construct a scale drawing to show the positions of the three towns.



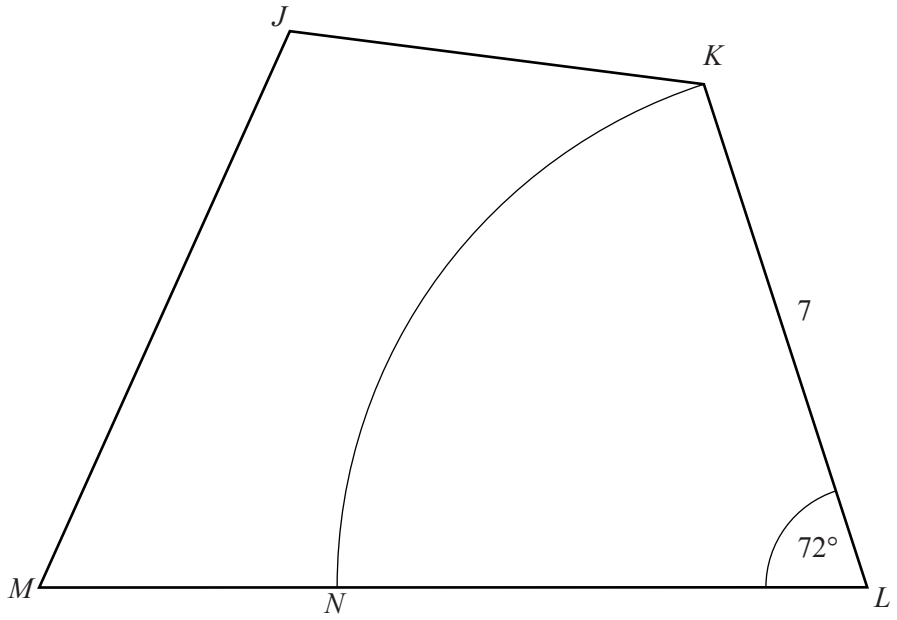

[3]

**(iii)** Measure the bearing of $B$ from $C$.

*Answer* ..... [1]

**(b)** *JKLM* is a quadrilateral with $KL = 7$ cm and $K\hat{L}M = 72°$.
*N* is the point on *LM* such that *KLN* is a sector of a circle, centre *L*.






**(i)** Calculate the area of the sector *KLN*.

*Answer* .....cm$^2$ [2]

**(ii)** Calculate the perimeter of the sector *KLN*.

*Answer* ..... cm [2]

**(iii)** On the diagram, construct the locus of points inside the quadrilateral *JKLM* which are

**I** 5 cm from *JM*,
**II** equidistant from *JK* and *KL*. [2]

**(iv)** The point *P* is inside *JKLM*,
less than 5 cm from *JM*,
nearer to *KL* than *JK*,
less than 7 cm from *L*.

Shade the region containing the possible positions of *P*. [1]

**9** A group of 80 music students recorded the time each spent practising last week.
The results are summarised in this table.

*Do not write in this margin*

| Time ($m$ minutes) | $0 < m \leqslant 20$ | $20 < m \leqslant 40$ | $40 < m \leqslant 60$ | $60 < m \leqslant 80$ | $80 < m \leqslant 100$ | $100 < m \leqslant 120$ |
|---|---|---|---|---|---|---|
| Frequency | 6 | 15 | 29 | 18 | 9 | 3 |

**(a)** Calculate an estimate of the mean.

*Answer* ..... minutes [3]

**(b)** Complete the cumulative frequency table below.

| Time ($m$ minutes) | $m = 0$ | $m \leqslant 20$ | $m \leqslant 40$ | $m \leqslant 60$ | $m \leqslant 80$ | $m \leqslant 100$ | $m \leqslant 120$ |
|---|---|---|---|---|---|---|---|
| Cumulative frequency | 0 | 6 | 21 | | | | 80 |

[1]

**(c)** For this part of the question use the graph paper opposite.

Using a scale of 1 cm to represent 10 minutes, draw a horizontal $m$-axis for $0 \leqslant m \leqslant 120$.
Using a scale of 2 cm to represent 10 students, draw a vertical axis for cumulative frequency from 0 to 80.
Using your axes draw a smooth cumulative frequency curve to illustrate the information.
[3]

**(d)** Use your graph to estimate

**(i)** the median,

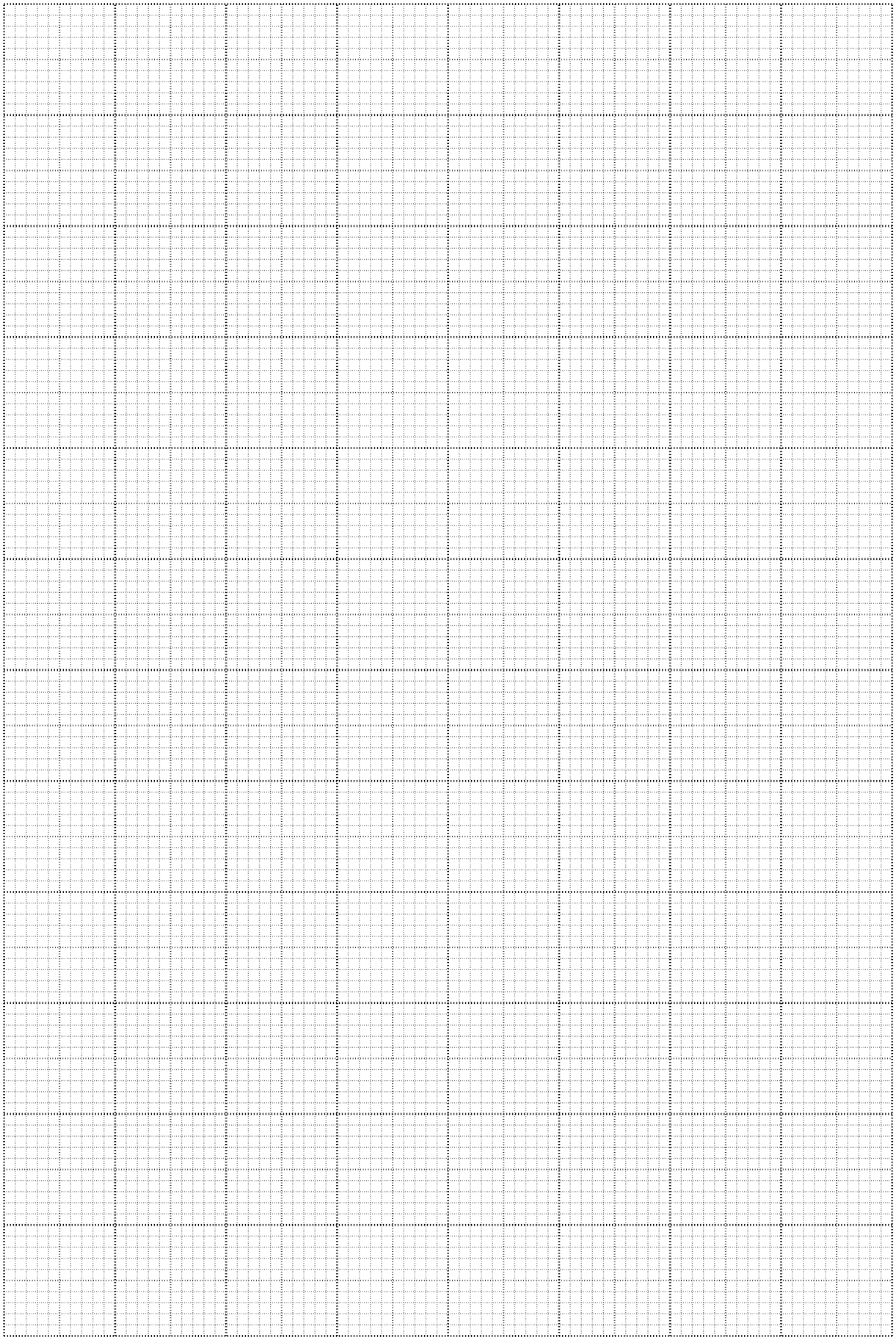
*Answer* ..... minutes [1]

**(ii)** the interquartile range,

*Answer* ..... minutes [2]

**(iii)** the probability that a student, chosen at random, practised for **more than** 75 minutes.

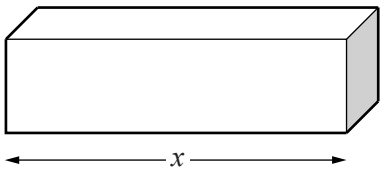
*Answer* ..... [2]



*Do not write in this margin*

**10**






A cuboid has a square cross-section, shown shaded in the diagram.
The length of the cuboid is $x$ cm.
The sum of the length of the cuboid and one of the sides of the square is 10 cm.

**(a)** Show that the volume of the cuboid, $y$ cm$^3$, is given by $y = x^3 - 20x^2 + 100x$.

[2]

**(b)** The table shows some values of $x$ and the corresponding values of $y$ for

$$y = x^3 - 20x^2 + 100x.$$

| $x$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| $y$ | 81 | 128 | 147 | 144 | 125 | 96 | | | 9 |

**(i)** Complete the table. [1]

**(ii)** On the grid opposite, plot the graph of $y = x^3 - 20x^2 + 100x$ for $1 \leqslant x \leqslant 9$. [3]

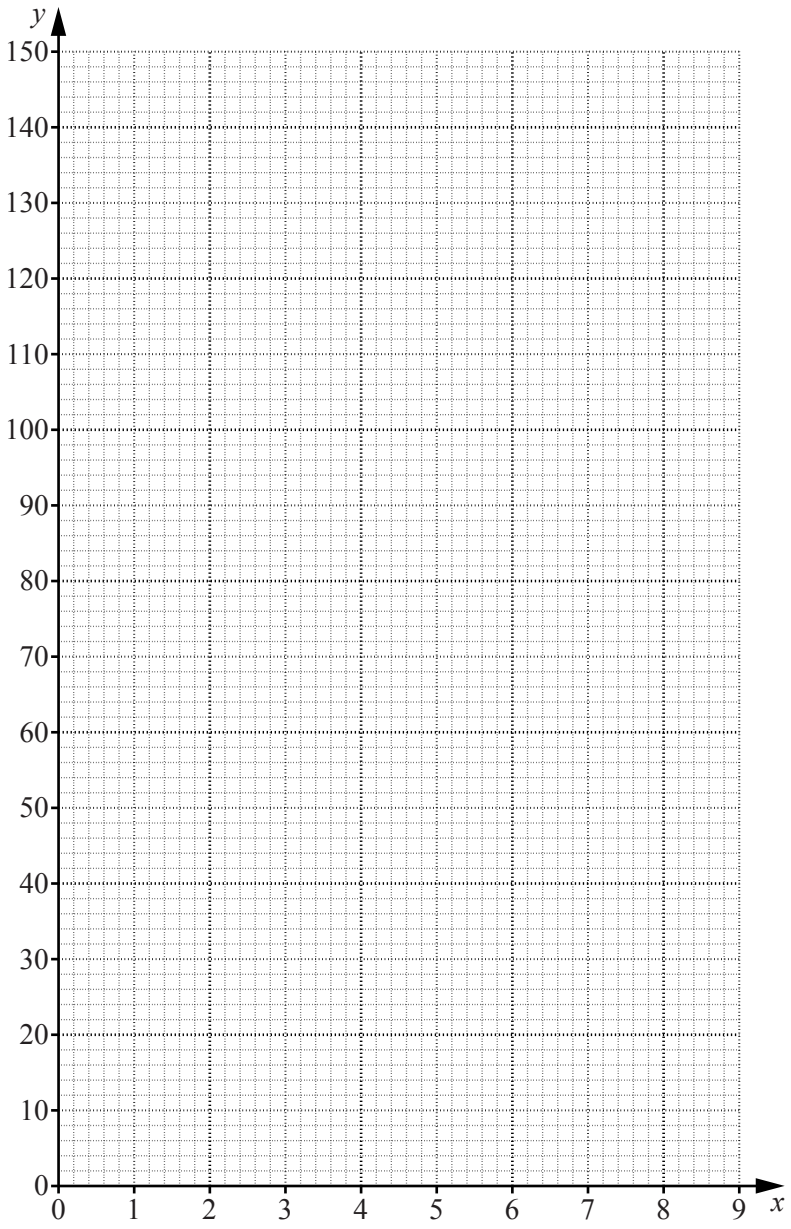
**(c)** Use your graph to find

**(i)** the maximum volume of the cuboid,

*Answer* ..... cm$^3$ [1]

**(ii)** the possible values of $x$ when the volume of the cuboid is 120 cm$^3$.

*Answer* $x =$ ..... or .....[2]




**(d) [The volume of a sphere = $\frac{4}{3}\pi r^3$]**

For this part of the question take π as 3.

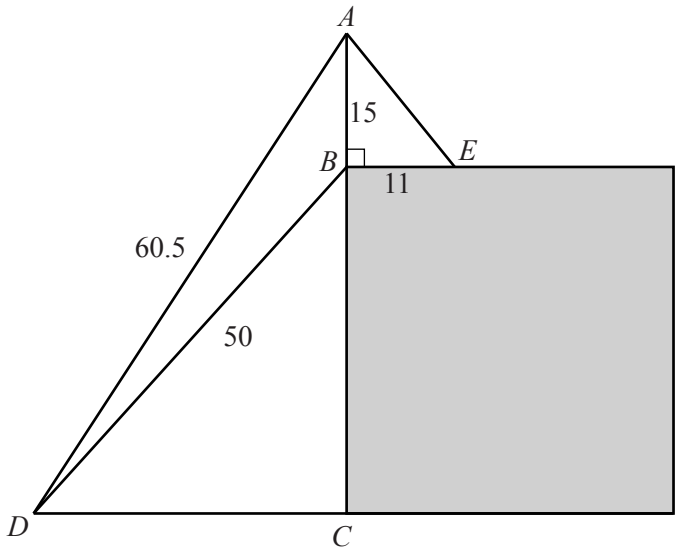
A sphere has a radius of $\frac{1}{2}x$ cm.

By drawing a suitable graph on the grid, estimate the value of $x$ when the sphere and the cuboid have the same volume.

*Answer* $x =$ ..... [3]

**11**






A vertical mast, $AB$, is 15 m tall and is attached to the top of a building at $B$. The top of the mast is attached to the roof of the building at $E$ using a wire. $A\hat{B}E = 90^{o}$ and $BE = 11$ m.

**(a) (i)** Calculate $AE$.

*Answer* .....m [2]

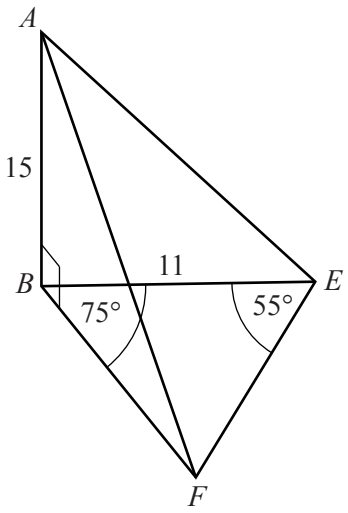
**(ii)** $D$ is a point on the ground such that $AD = 60.5$ m and $BD = 50$ m.

Calculate $A\hat{D}B$.

*Answer* .....[4]

**(b)**






The top of the mast is also attached to the roof of the building at $F$ using a wire.
$F\hat{B}E = 75°$, $B\hat{E}F = 55°$ and $A\hat{B}F = 90°$.

**(i)** Calculate $B\hat{F}E$.

*Answer* ..... [1]

**(ii)** Calculate $FB$.

*Answer* ..... m [3]

**(iii)** Calculate the angle of depression of $F$ from the top of the mast.

*Answer* ..... [2]

**12** **(a)** $\mathbf{A} = \begin{pmatrix} -3 & 6 \\ -2 & 2 \end{pmatrix}$ $\mathbf{B} = \begin{pmatrix} -1 & 0 \\ 1 & -2 \end{pmatrix}$

Do not write in this margin

Find

**(i)** $\mathbf{A} + 2\mathbf{B}$,

*Answer* $\begin{pmatrix} & \\ & \end{pmatrix}$ [1]

**(ii)** $\mathbf{A}^{-1}$.

*Answer* $\begin{pmatrix} & \\ & \end{pmatrix}$ [2]

**(b)** Mark and Luke spend three days training for a cycling event.
Mark cycles at an average speed of 24 km/h on the first two days and 26 km/h on the third day.
Luke cycles at an average speed of 25 km/h on the first day, 24 km/h on the second day and 27 km/h on the third day.
They each cycle for 1 hour on the first day and increase their cycling time by $\frac{1}{2}$ hour each day.

This information is represented by the matrices **P** and **Q** below.

$$\mathbf{P} = \begin{pmatrix} 24 & 24 & 26 \\ 25 & 24 & 27 \end{pmatrix} \qquad \mathbf{Q} = \begin{pmatrix} 1 \\ m \\ n \end{pmatrix}$$

**(i)** Find $m$ and $n$.

*Answer* $m$ = ..... $n$ = .....[1]

**(ii)** Find **PQ**.

*Answer* [2]

**(iii)** Calculate the difference between the numbers in the matrix **PQ** and explain what this number represents.

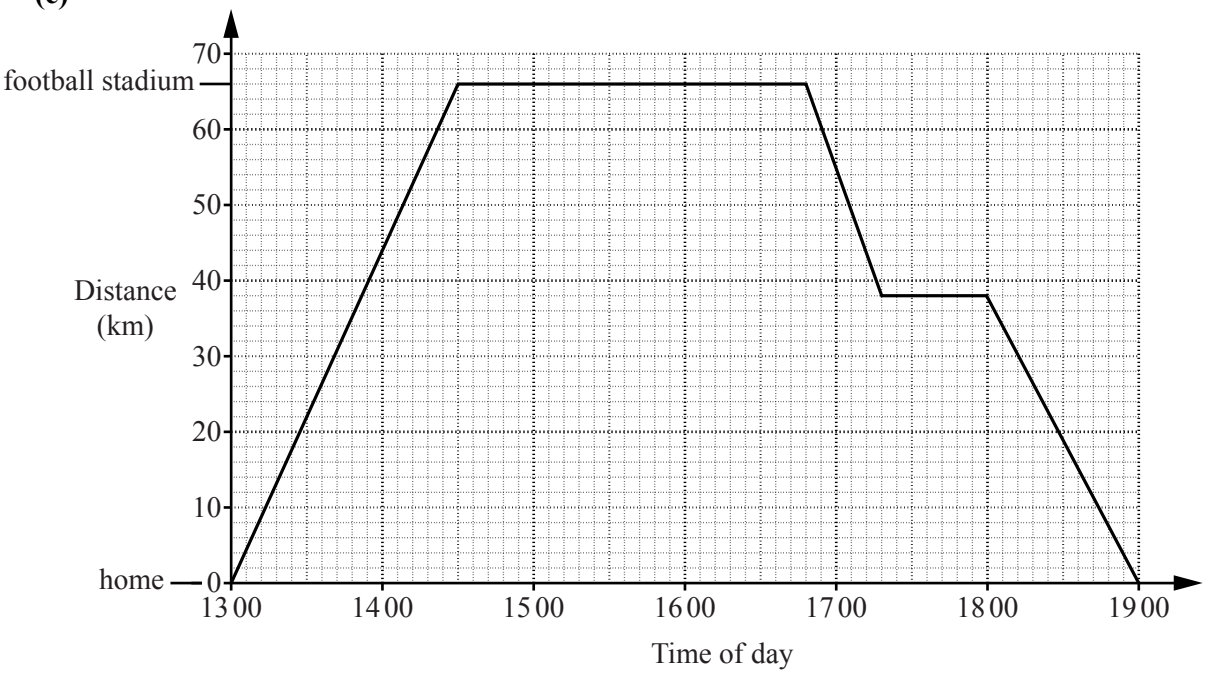
*Answer* Difference is ..... and this number represents .....

.....[2]

**Please turn over for the rest of this question**

*Do not write in this margin*

**(c)**




The travel graph represents Sahid's journey to and from a football match.
He travels from home straight to the football stadium and watches the match.
On his way home he stops at a café.

**(i)** How many minutes does Sahid spend at the football stadium?

*Answer* ..... minutes [1]

**(ii)** What is his speed, in kilometres per hour, on his journey to the football stadium?

*Answer* .....km/h [1]

**(iii)** What is the distance between the football stadium and the café?

*Answer* .....km [1]

**(iv)** Between which two places does he travel the fastest?

*Answer* ..... and .....[1]

*Do not write in this margin*